Impact of the Casimir Force on Movable-Dielectric RF MEMS Varactors

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***Abstract*-An analysis of factors affecting the tuning range of movable-dielectric RF MEMS varactors, in particular, lateral and vertical spring stiffness and plate-slab gap asymmetry, is conducted. The analysis is based upon simple analytical models for spring and electrostatic forces, and on Lifshitz's formula for molecular interaction forces. It is found that, due to the large sensitivity of Casimir forces to asymmetry, these are very likely to play a greater role than spring vertical stiffness, in limiting the varactor tuning range.**

I. INTRODUCTION

To overcome the performance-limiting mechanisms in the parallel-plate MEM varactor, namely, support spring resistance-limited quality factor, high actuation voltage, and pull-in-limited tuning range, a movable-dielectric varactor, Fig. 1, was proposed and demonstrated by Yoon and Nguyen [1].

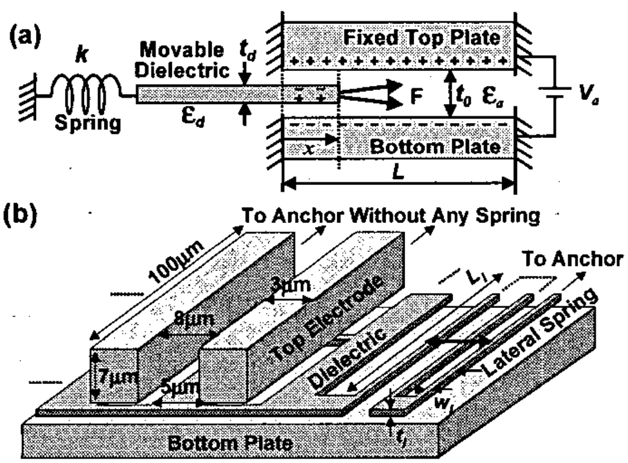

Figure 1. Movable-dielectric MEM varactor concept (a) Conceptual schematic. (b) Actual implementation. (Courtesy of Dr. J.B. Yoon, KAIST.)

In this device a voltage V, applied between rigid top and bottom plates, pulls *laterally* a dielectric slab of permittivity ε_d into the inter-plate gap, t_0, with an *electrostatic* force F given by (1),

$$F = \frac{L_1 t_d \varepsilon_a (\varepsilon_d - \varepsilon_a)}{2t_0[(t_0 - t_d)\varepsilon_d + \varepsilon_a t_d]} \cdot V^2, \qquad (1)$$

where L_1 is the width of the plates, and t_d is the dielectric slab thickness. As the dielectric is pulled into the gap, the capacitance changes as given by (2),

$$C = \frac{\varepsilon_a L L_1}{t_0} + \left[\frac{\varepsilon_a \varepsilon_d}{(t_0 - t_d)\varepsilon_d + \varepsilon_a t_d} - \frac{\varepsilon_a}{t_0}\right] L_1 x \qquad (2)$$

where L is the length of the plates.

While numerical simulations of capacitance-voltage tuning predicted a maximum change of 47% for a nitride dielectric slab, Yoon and Nguyen [1] discovered that measured results yielded a capacitance change of only ~15%. Investigations on the nature of this discrepancy suggested that a design oversight may have rendered the vertical spring constant of the dielectric slab more than 10 times smaller than the lateral one and that, under these conditions, they stated, the voltage applied across the slab, with asymmetrical dielectric-to-metal gaps above and below it, gives rise to a finite electrostatic *vertical* force. As a result, the dielectric slab experienced limited lateral travel as it soon, upon actuation, deflected towards, and came into contact with, one of the rigid plates, thus precluding further lateral motion. While this explanation is plausible, the fact that the gap between the rigid metallic plates is just $t_0 = 0.5\mu m$, suggests that quantum mechanical phenomena, in particular, Casimir forces [2, 3], might also be responsible for effecting the observed vertical deflection of the dielectric slab and, thus, should not be discarded. The Casimir forces in this case arise from the polarizability of the dielectric slab caused by the vacuum electromagnetic (EM) modes delimited by the narrowly-spaced rigid metallic plates.

In this paper, we study the relative magnitude of the various forces exerted on the dielectric slab with a view towards elucidating the importance of the Casimir force.

II. FORCES ON DIELECTRIC SLAB

In general, the *intruding* dielectric slab experiences the following *net* force components: (1) A *lateral* force which embodies the balance between the electrostatic force pulling the dielectric slab into the gap, due to the inhomogeneity in the energy density in the inter-plate air-gap and in the dielectric slab [4], and the opposing force due to the spring constant of the dielectric suspension. The resultant lateral

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displacement is obtained from the equilibrium between these two forces, namely,

$$\frac{L_1 t_d \varepsilon_a(\varepsilon_d - \varepsilon_a)}{2t_0[(t_0 - t_d)\varepsilon_d + \varepsilon_a t_d]} \cdot V^2 - k_L x = 0. \quad (3)$$

(2) A vertical force which embodies the balance between the vertical electrostatic forces between the rigid plates and the dielectric slab [5], the opposing spring force due to the finite vertical stiffness of the dielectric slab, and the net Casimir force between the rigid plates and the dielectric slab. The resultant vertical displacement is obtained from the equilibrium among these four forces, namely,

$$\frac{\varepsilon_a L_1 x V^2}{2(t_{a1} - \Delta y)^2} - \frac{\varepsilon_a L_1 x V^2}{2(t_{a2} + \Delta y)^2} - k_V \Delta y + F_{Casimir} = 0. \quad (4)$$

In equation (4) we assume that the gap between the slab and top rigid plate t_{a1}, is smaller than that between the slab and the bottom rigid plate t_{a2}, so that upon voltage application the slab deflects upwards a distance Δy. The Casimir force per unit area between the rigid plates and the dielectric is given by Lifshitz's formula [6], equation (5), in the limiting case for "large" separations $t_0 >> \lambda_0$, where λ_0, the characteristic of the absorption spectra of the metal is taken as the plasma frequency.

$$F(z) = \frac{hc}{32\pi^2 z^4 \sqrt{\varepsilon_{30}}} \int_0^\infty dx \int_1^\infty \frac{x^3}{p^2} \left\{ \left[\frac{s_{10}+p}{s_{10}-p} \frac{s_{20}+p}{s_{20}-p} e^x - 1 \right]^{-1} + \left[\frac{s_{10}+(p\varepsilon_{10}/\varepsilon_{30})}{s_{10}-(p\varepsilon_{10}/\varepsilon_{30})} \frac{s_{20}+(p\varepsilon_{20}/\varepsilon_{30})}{s_{20}-(p\varepsilon_{20}/\varepsilon_{30})} e^x - 1 \right]^{-1} \right\} dp \quad (5)$$

In (5) h is Planck's constant, c is the speed of light, $s_{i0} = \sqrt{(\varepsilon_{i0}/\varepsilon_{30}) - 1 + p^2}$, and z is the distance between bodies 1 and 2, characterized by electrostatic dielectric constants $\varepsilon_{10} = \varepsilon_1(\omega = 0)$, $\varepsilon_{20} = \varepsilon_2(\omega = 0)$, respectively, and separated by body 3, with $\varepsilon_{30} = \varepsilon_3(\omega = 0)$. In the present device, Fig. 1, bodies 1, 2, and 3 are realized by the rigid metallic plates, the dielectric slab, and vacuum ($\varepsilon_{30} = 1$) respectively. Therefore, utilizing Drude's model [7] for the metallic dielectric constant, namely,

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad (6)$$

and taking the limit $\varepsilon_{io} = \lim_{\omega \to 0} \varepsilon_i(\omega)$, we obtain the force of *attraction* between a rigid metallic plate and the dielectric slab as,

$$F(z) = \frac{hcL_1 x}{32\pi^2 z^4} \int_0^\infty dx \int_1^\infty \frac{x^3}{p^2} \left\{ \left[\frac{\varepsilon_d + p}{\varepsilon_d - p} e^x - 1 \right]^{-1} + \left[\left(\frac{1+p}{1-p} \right)^2 e^x - 1 \right]^{-1} \right\} dp \quad (7)$$

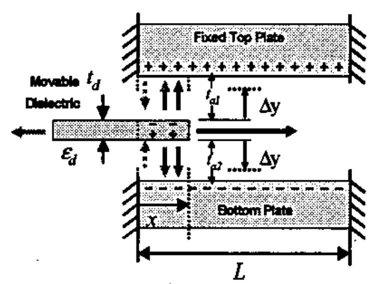
Using $\varepsilon_d = 7.5$ for the dielectric constant of silicon nitride, the slab material in the present device, substituting in the values of h and c, and carrying out the integrations, (7) reduces to,

$$F(z) = 2.992 \times 10^{-25} \frac{L_1 x}{z^4} \quad (8)$$

With this, the *net* Casimir force exerted on the slab is given by,

$$F_{Casimir} = 2.992 \times 10^{-25} L_1 x \left[\frac{1}{(t_{a1} - \Delta y)^4} - \frac{1}{(t_{a2} + \Delta y)^4} \right]. \quad (9)$$

The overall set of forces acting on the dielectric slab may be depicted as in Fig. 2.



← $F_{Spring\_1} = -k_L x$	↕ $F_{Spring\_2} = -k_V \Delta y$	↑ $F_{Electrostatic\_1} = \frac{\varepsilon_a L_1 x V^2}{2(t_a - \Delta y)^2}$ ↓ $F_{Electrostatic\_2} = \frac{\varepsilon_a L_1 x V^2}{2(t_a + \Delta y)^2}$
↑ $F_{Casimir\_1} = \frac{2.992\times10^{-25} L_1 x}{(t_{a1} - \Delta y)^4}$ ↓ $F_{Casimir\_2} = \frac{2.992\times10^{-25} L_1 x}{(t_{a2} + \Delta y)^4}$	→ $F_{Lateral} = \frac{L_1 t_d \varepsilon_a(\varepsilon_d - \varepsilon_a)}{2t_0[(t_0 - t_d)\varepsilon_d + \varepsilon_a t_d]} \cdot V^2$	

Figure 2. Forces acting on dielectric slab.

III. STUDY OF CAPACITOR TUNING RANGE

Ideally, the change in capacitance, or tuning range, of the movable-dielectric varactor is given by,

$$\Delta C = \frac{1}{\frac{\varepsilon_d t_0}{(\varepsilon_d - \varepsilon_a) t_d} - 1} \cdot \frac{x}{L}. \quad (10)$$

Thus, for a slab with dielectric constant much greater than that of the vacuum, the tuning range is a function of inter-plate gap, slab thickness, plate length, and distance the slab intrudes into capacitor, x. This latter is dictated by the applied voltage and the lateral spring constant of the slab, as per (3). Fig. 3 shows the change in capacitance versus applied voltage for various values of lateral spring stiffness, indicating the obvious fact that, the softer the spring, the greater the slab displacement.

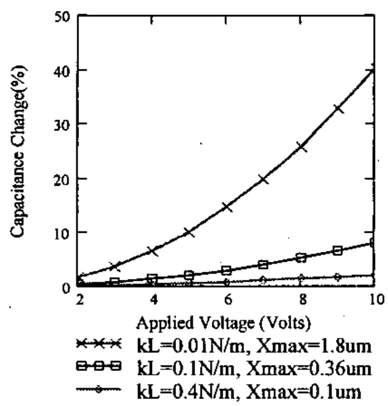


Figure 3. Capacitance change versus applied voltage for movable-dielectric varactor at a various lateral spring constant, k_L, values. In this model, k_L=0.01N/m produces a maximum capacitance change of ~40%. Parameters: ε_d=7.5, ε_a=1, t_0=1.1*um*, t_{a1}=0.2*um*, t_{a2}=0.3*um*, t_d=0.6*um*.

Equation (3) assumes either that the slab experiences no vertical force or that its stiffness is infinite. Obviously, the real slab must have a finite vertical stiffness which is, in fact, given by,

$$k_{Vertical} = \left(\frac{t_d}{L_d}\right)^3 \times W_d E \times \frac{1}{N}, \quad (11)$$

where t_d, L_d, and W_d are thickness, length, and width of the meanders (see Fig. 1), E is the slab material's Young's modulus, and N is the number of meanders. Furthermore, as indicated by (4), (9) and Fig. 2, vertical forces may arise from *asymmetry* in the air gaps, above and below the slab, which gives rise to imbalances in the electrostatic and Casimir forces between the slab and the top and bottom rigid metallic plates, thus eliciting a vertical displacement obeying (4). Clearly, forces causing a vertical displacement component of the slab, as it moves laterally, would eventually lead it to prematurely touch either of the rigid plates and, thus, reduce the tuning range. An interesting question that arises in this case is this: Which of the vertical forces present due to the gap asymmetry, namely, the electrostatic and the Casimir forces, is responsible for the reduced tuning range? This question is interesting because, if it turns out that one can discard the electrostatic force, then one can conclude that the reduced tuning range is the result of the quantum mechanical zero-point fluctuations of the electromagnetic field vacuum, i.e., of the Casimir effect [2], thus elucidating a clear manifestation of this effect in these devices.

To address this question we examine the effects of the vertical spring constant, k_V, and of the top and bottom air-gap asymmetry about dielectric slab, on the observed capacitance change, displayed in Fig. 4.

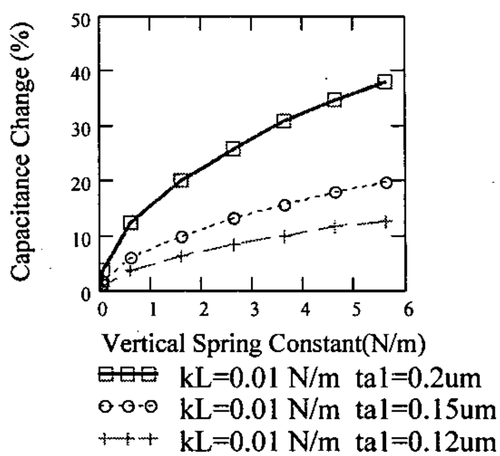


Figure 4. Maximum capacitance change versus vertical spring constant as a function of top and bottom air-gap asymmetry about dielectric slab. Parameters: ε_d=7.5, ε_a=1, t_0=1.1*um*, t_d=0.6*um*, $t_{a1}+t_{a2}$=0.5*um*.

It can be surmised from Fig. 4, that an increase in asymmetry from 33% (t_{a1}=0.2um/t_{a2}=0.3um), to 57% (t_{a1}=0.15um/t_{a2}=0.35um) causes a reduction in tuning range of 18%, whereas an increase in asymmetry from 33% to 68% (t_{a1}=0.12um/t_{a2}=0.38um) causes a tuning range reduction of 25%. Thus, the greater the asymmetry, the greater the reduction in tuning range. On the other hand, a reduction in the ratio of vertical to lateral spring constant k_V/k_L from 560 to 0.4 causes corresponding reductions in tuning range of 38% (t_{a1}=0.2um/t_{a2}=0.3um), 20% (t_{a1}=0.15um/t_{a2}=0.35um), and 13% (t_{a1}=0.12um/t_{a2}=0.38um). These results suggest that the effect vertical-to-lateral stiffness ratio may be negligible compared to that of asymmetry in that, it takes a relatively large change in the former, i.e., several hundred, compared to a smaller change in asymmetry, i.e., ~0.05um, to cause a comparable change in tuning range. The reason for this appears to be apparent from an examination of the ratio of the *net* Casimir force to the *net* electrostatic force acting on the slab, Fig. 5. The figure shows that the greater the asymmetry, the greater than unity is this ratio. This can be understood from the dependence on the inverse plate-slab gap distance. In particular, whereas the electrostatic force varies as $F_{Electrostatic} \propto z^{-2}$, the Casimir force varies as

$F_{Casimir} \propto z^{-4}$, thus the latter rises faster at the asymmetry increases. This inverse fourth-power dependence of the Casimir force on the gap, furthermore, makes it more sensitive to changes in it than the otherwise first-power dependence of the spring force on stiffness.

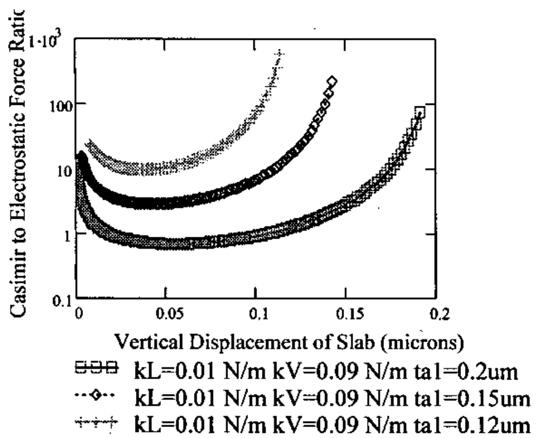


Figure 6. Ratio of Casimir force to electrostatic as a function of top and bottom air-gap asymmetry about dielectric slab. Parameters: ε_d=7.5, ε_a=1, t_0=1.1*um*, t_d=0.6*um* $t_{a2}+t_{a2}$=0.5*um*.

IV. Summary and Conclusions

The impact of lateral and vertical spring stiffness, and gap asymmetry, on the tuning range of the *intrinsic* (i.e., neglecting fringing fields) movable-dielectric RF MEMS varactor has been studied. It was found that, due to the inverse fourth-power dependence of the Casimir force on the capacitor plate-slab gap, this force plays a dominant role over electrostatic forces and vertical spring (stiffness) forces when the degree of asymmetry is high and submicron gaps are employed. This phenomenon elucidates the existence of a potential trade-off in the design of these devices, namely, between tuning range and actuation voltage, not unlike that found in parallel-plate variable-gap varactors [1], [5], which the movable-dielectric varactor is intended to improve upon.

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