A Theoretical Study of Nanoelectromechanical Quantum Tunneling Frequency Multipliers

Héctor J. De Los Santos
NanoMEMS Research, LLC
Irvine, California, USA
hjd@nanomems-research.com

***Abstract*— Frequency multipliers (FMs) are ubiquitous signal processing circuits in communication systems. Traditionally, this circuit is based on exploiting the nonlinearities found in semiconductor devices, for example, bipolar junction transistors and Schottky diodes. We explore an FM concept based on the coexistence of spring, electrostatic, and Casimir forces, and tunneling. In particular, we provide, for the first time, a theoretical analysis of a nanoelectromechanical quantum tunneling FM. The physics behind the nature of the frequency multiplication mechanism will be elucidated, together with a discussion of circuit modeling, applications and limitations. SPICE simulations are presented.**

Keywords-nanoelectromechanical; microwave; circuit; frequency multiplier; Casimir; quantum mechanical.

I. INTRODUCTION

Frequency multipliers (FMs) are ubiquitous signal processing circuits in communication systems. Traditionally, this circuit is based on exploiting the nonlinearities found in semiconductor devices, for example, bipolar junction transistors and Schottky diodes. With the emergence of nanoelectromechanical systems fabrication techniques, the ability to take advantage of the coexistence of electronic and mechanical functions that exploit quantum effects, to realize potentially superior performance is only limited by our imagination. In this frame of mind, in particular, we aim at exploring how this convergence might give rise to a "new electronics." This paper provides, for the first time, a theoretical analysis of a nanoelectromechanical quantum tunneling frequency multiplier. In particular, the physics behind the nature of the frequency multiplication mechanism will be elucidated, together with a discussion of circuit modeling, applications and limitations.

The electrostatically-actuated cantilever beam may be considered as one of the fundamental elements in the MEMS/NEMS device arsenal [1]. It finds applications in MEM switches, varactors, and resonators, to name a few. When configured so that it resembles a field-effect transistor (FET) in which the tip of the cantilever beam behaves as a tunneling contact whose deflection is controlled by the gate-beam potential difference, the device has been shown by McCord, Dana and Pease [2] to implement a micromechanical tunneling transistor (MMTT), see Fig. 1.

With a tunneling current given by [2, 3],

$$I = V_{ds}\, K\, \exp(-1.025\sqrt{\phi}\, w) \qquad (1)$$

where ϕ and w are the tunneling barrier height and width, respectively, the MMTT transconductance is given by [2],

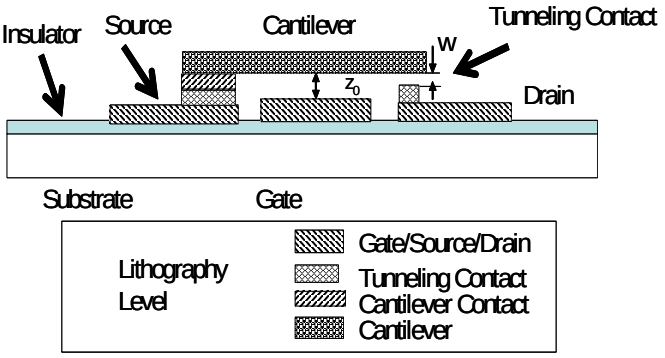


Figure 1. Cross-section of MMTT showing basic device structure as well as lithography levels required for fabrication.

$$G_I = \frac{\partial I}{I \partial V_g} = \frac{1}{I}\frac{\partial I}{\partial z}\frac{\partial z}{\partial V_g} = \frac{1.025\sqrt{\phi} K A \varepsilon V_g}{k z_0^2 \sqrt{1 - 4A\varepsilon V_g^2 / k z_0^3}} \qquad (2)$$

where A is the gate area, V_g is the gate potential, Vds is he source-drain voltage, k is the cantilever spring constant, ε is the permittivity between gate and beam, and z_0 is the gate-beam gap. Assuming I=1nA, $\partial I/\partial z = 10nA \;\; nm^{-1}$, and $\partial z/\partial V_g = 50nm \;\; V^{-1}$, McCord *et. al.* [2] estimated that a transconductance *per unit current* of 500 V^{-1} should be achievable [2]. This transconductance, being 12.5 times the theoretical maximum value exhibited by a bipolar transistor at room temperature, led the authors to conclude that the MMTT

has great potential for extremely high performance low-cost, low-power applications [2].

The MMTT operation relies on *pre-pull-in* beam actuation, to achieve fine tunneling tip displacements, of the order of nanometers for nA-type tunneling currents, and low voltages to prevent electrical breakdown between the gate and the cantilever and damage to the tunneling junction [2].

In this paper, we study the operation of the MMTT as a *frequency multiplier* (FM). In particular, since the gate-beam gap is well below 1um, we will study the effect of the presence of Casimir forces [4]-[5] in determining the modulation of the tunneling barrier width, and its concomitant effect on the frequency spectrum.

II. MMTT Frequency Multipliers (XMMTT)

A. Prior-Art Tunneling Current FMs

Frequency multipliers exploiting tunneling-controlled transport have been previously demonstrated [6]. These were based on exploiting the tunneling collector current in double-heterostructure bipolar transistors (DHBTs) with an incompletely-graded base-collector (BC) conduction band discontinuity captured by [7],

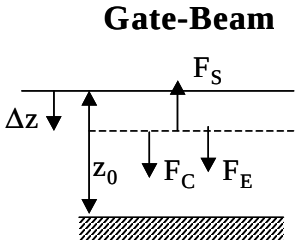
$$I_C \sim V_{BE}^2 \exp(-b / V_{BE}) \qquad (3)$$

where V_{BE} is the base-emitter voltage and *b* embodies the tunneling parameters [8] of the BC junction. One of the key properties of DHBTs, namely, their high cutoff frequency (f_T), was responsible for endowing DHBT-based FMs with high DC efficiencies, since for a given output harmonic *n*, of an input frequency f_0, the current conversion efficiency is approximately f_T/nf_0 [7]. While DHBT-based FMs exhibit an improved DC efficiency, due to the non-exponential collector tunneling current, (3), most of the harmonic power is concentrated in the lower harmonics. This is not necessarily undesirable, however, because, as is well known, higher multiplication factors n are associated higher noise and tend to be avoided in many circumstances. Because of its high DC efficiency at low-order n, DHBT-based FMs are now standard in many systems. Obviously, since the DHBTs are rather exotic III-V compound semiconductor devices which require sophisticated fabrication techniques, in particular, molecular beam epitaxy (MBE), there is a cost associated with them, in contrast to, e.g., silicon devices. MMTTs, being fabricated using simple surface micromachining techniques, offer the potential to realize passive FMs with enhanced loss properties.

B. Tunneling Barrier Width Modulation

In an MMTT the nominal barrier width (w) is reduced by the deflection Δz of the beam, see Fig. 1, which results from the application of the voltage $V = V_g + V_{ds}$, the sum of the gate-beam and source-drain voltages. Δz is obtained, in turn, from an equation for the balance of the forces determining the cantilever beam equilibrium position, (4), namely, the electrostatic and spring [1], and Casimir [4, 5], forces, Fig. 2.

$$\frac{\pi^2 \hbar c A}{240\,(z_0 - \Delta z)^4} + \frac{\varepsilon A V^2}{2(z_0 - \Delta z)^2} - k_S \Delta z = 0 \qquad (4)$$

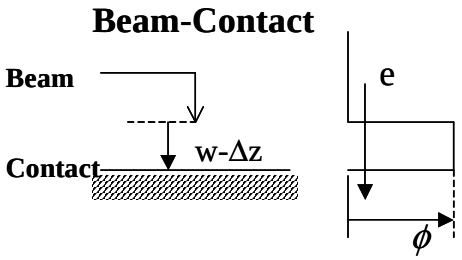


$$F_S = k_S \Delta z \quad F_E = \frac{\varepsilon A V^2}{2(z_0 - \Delta z)^2} \quad F_C = \frac{k_C}{(z_0 - \Delta z)^4}$$

For equilibrium: $|F_S| = |F_E + F_C| \quad k_C = \frac{\pi^2 \hbar c A}{240}$

Beam is Actuated by:
- Coulomb electrostatic force
- Casimir force

(a)



Transport Mechanism: Tunneling

$$I_{Tun} = V_{ds} K \exp(-1.025 \sqrt{\phi}\, w)$$

ϕ is the tunneling barrier energy height
w is the tunneling barrier width

(b)

Figure 2. MMTT Frequency Multiplier Physics. (a) Gate-Beam actuation: $\hbar$ is Planck's constant divided by 2π, *c* is the speed of light. (b) Beam-Contact tunneling.

The dominance of the various actuation force components changes as a function of the beam displacement, as shown in Fig. 3. In particular, it can be surmised from Fig. 3 that, at small beam displacements (large remaining beam-gate

distance), the spring force is dominant over both the electrostatic and Casimir forces, and the electrostatic force, in turn, is dominant over the Casimir force. However, at large beam displacements (small remaining beam-gate distance), the Casimir force takes over and becomes the dominant force.

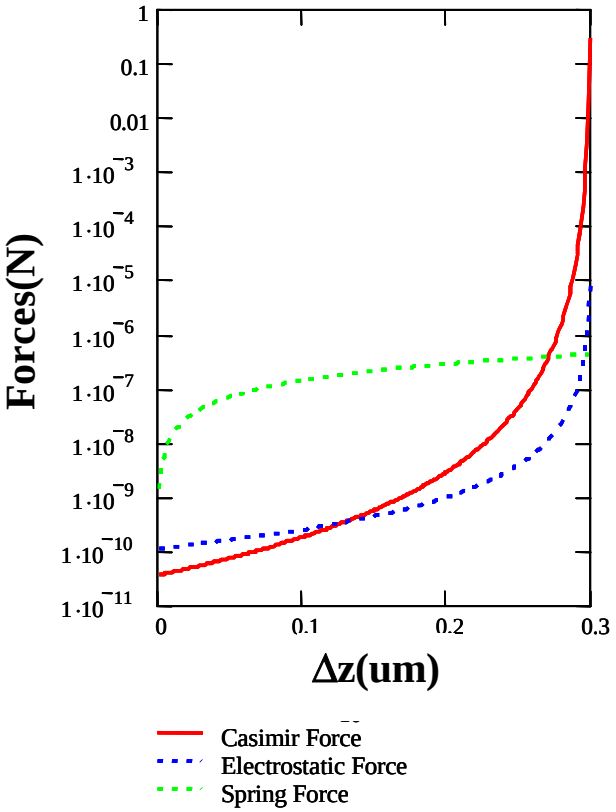


Figure 3. Typical relative variation of Casimir, electrostatic and spring forces in MMTT. Assumed geometrical device parameters: Beam width, length, thickness W=15um, L=15um, H=0.2um; k=1.422N/m.

The displacement Δz as a function of the applied voltage V is obtained by expressing (4) in powers of Δz. The resulting equation is a quintic, i.e., possesses five roots. Of the five roots obtained, Fig. 4, one is real and four are complex. The real root (Root 1), in particular, captures the displacement of the beam under the applied actuation voltage. Notice that pull-in occurs at the point at which the displacement (Root 1) reaches about 100nm, that is, at about one-third the original beam-gate gap of z_0= 300nm. Clearly, the beam must be operated at pre-pull-in biases to avoid crashing the fine tunneling tip on the contact and, in particular, its displacement should never go beyond the nominal beam tip-to-contact distance *w*., i.e., $w - \Delta z$ should never be zero.

C. XMMTT Circuit Model

The device properties of the XMMTT may be modeled by the circuit model shown in Fig. 5. In essence, since the gate-beam gap, z_0 is modulated by the applied gate-beam voltage, we model the input as a variable capacitor, CG, given by (5).

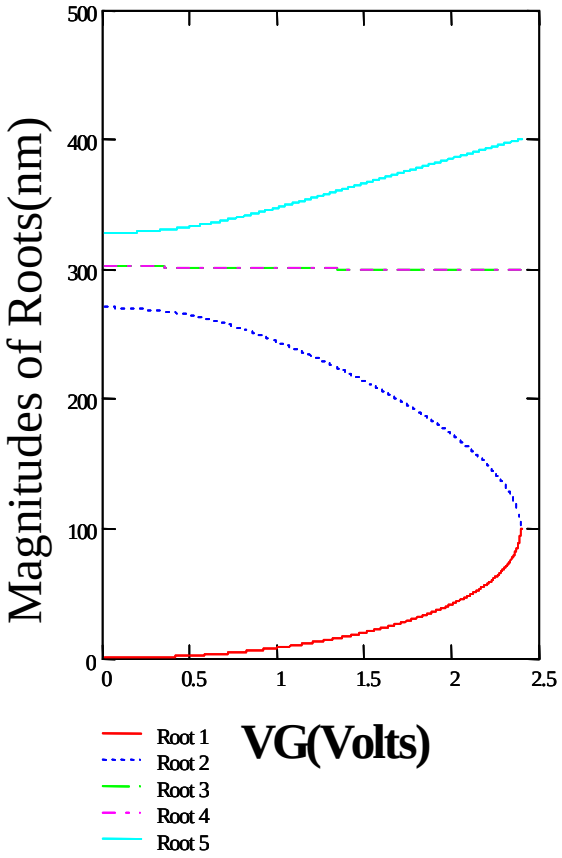


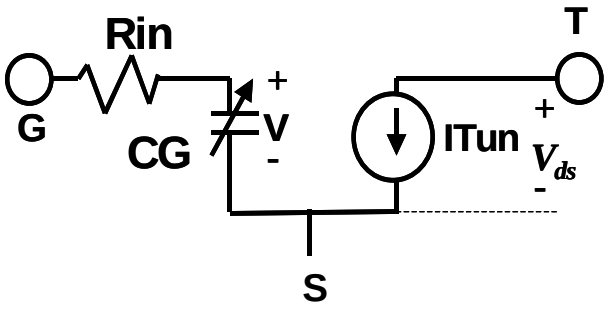
Figure 4. Magnitude of roots as a function of applied actuation voltage. Assumed geometrical device parameters: Beam width, length, thickness W=15µm, L=15µm, H=0.2µm; k=1.422N/m; z_0=0.3µm.

$$CG = \frac{\varepsilon_0 A}{z_0 - \Delta z(V)} \tag{5}$$

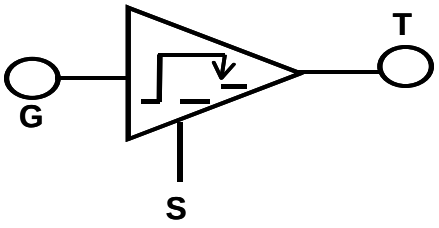
The voltage across this input varactor causes a displacement Δz, which, in turn, controls the tip-contact barrier width, w, and thus modulates the beam-drain tunneling current, given by (6).

$$I_{Tun} = V_{ds} K \exp(-1.025\sqrt{\phi}(w - \Delta z)) \tag{6}$$

In addition, since the tunneling current is proportional to the applied source-drain voltage, it is modeled as a voltage-controlled current source, *ITun* between drain and source. A resistor Rin in series with the gate is added. The complete circuit model is shown in Fig. 5, and a plot of ITun versus barrier width is shown in Fig. 6.



(a)



(b)

Figure 5. (a) XMMTT Circuit Model. (b) XMMTT symbol

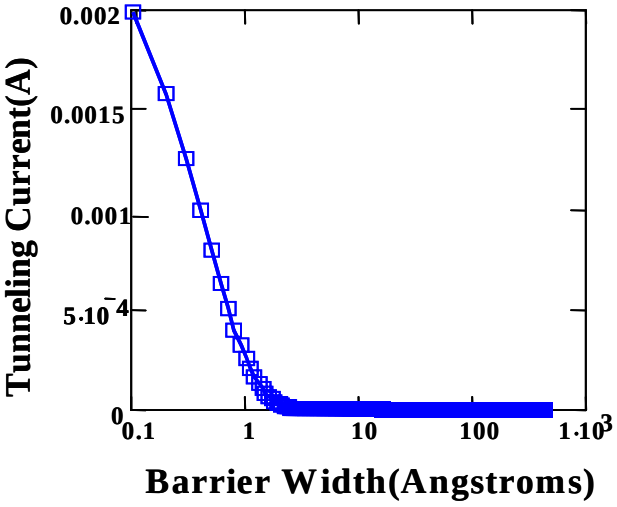


Figure 6. Tunneling current versus barrier width.

D. XMMTT Transient Response

The above circuit model was implemented in the circuit simulator SPICE and utilized to configure the circuit shown in Fig. 7. The maximum *input* frequency capability of the XMMTT is limited by the beam resonance frequency, $f_0 = (1/2\pi)\cdot\sqrt{k/m}$, where *k* is its stiffness and *m* its mass [1]. The selection of a highly stiff material and reduced mass beam offers a path to higher frequencies. Roukes [9], for example, has already demonstrated *nanoresonators* with frequencies exceeding 1GHz.

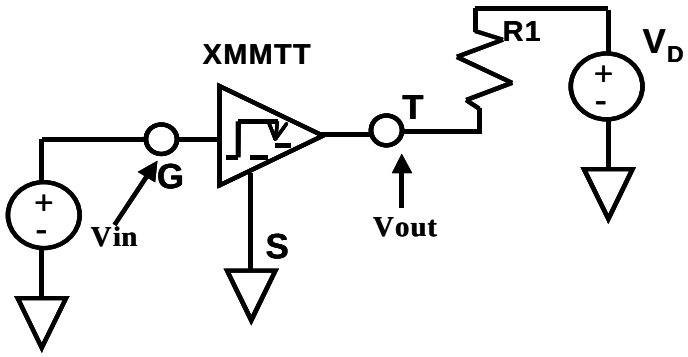


Figure 7. XMMTT frequency multiplier circuit. The following device parameters were assigned: w=422Å, V_D=2V, $z_0 = 0.3\mu m$, $\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$, $A = 225 \times 10^{-12}\ m^2$, $\phi = 5\ eV$, $K = 12.5 \cdot 10^{-4}$, $R1 = 1M\Omega$. Assuming a polysilicon beam, with stiffness k=1.422N/m, and mass density $\rho = 2331\ Kg/m^3$, the beam resonance frequency would be ~0.6MHz.

The response time and frequency domain responses to a 0.5 MHz sinusoidal input waveforms are shown in Figs. 8 and 9, respectively.

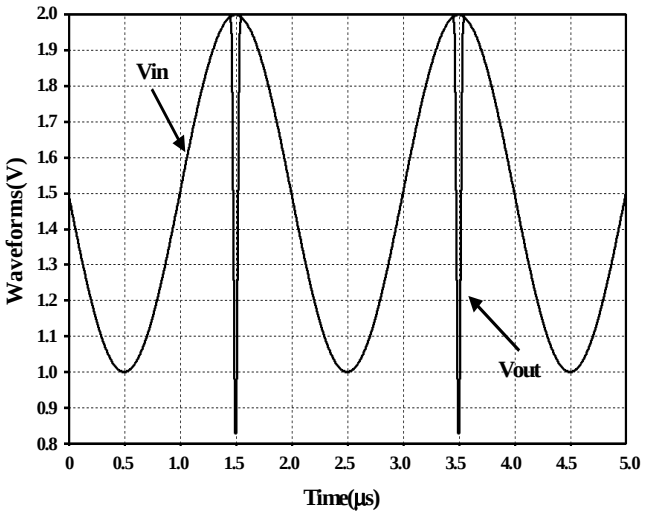


Figure 8. Waveforms of XMMTT frequency multiplier driven at 0.5 MHz.

This response shows sharp output voltage spikes, which may be understood as follows. At low input voltages, Vin, the tip-contact distance is so large that the tunneling current is virtually suppressed. This negligible current persists until the amplitude of Vin is such that the beam has moved to a tip-contact distance within a few Angstroms. At this point, the exponential nature of tunneling current causes the current to sharply increase, Fig. 6. This property of the XMMTT is what is exploited for frequency multiplication. Fig. 9 shows the

output spectrum, where the 42nd harmonic has amplitude of 10% the fundamental.

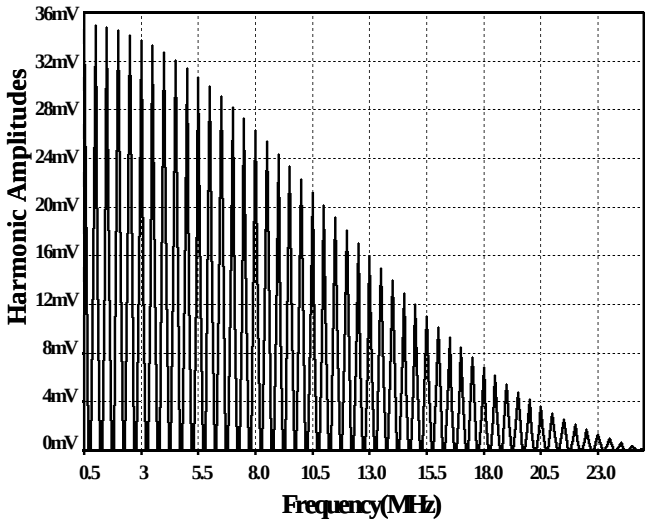


Figure 9. Output spectrum of XMMTT frequency multiplier driven at 0.5 MHz.

E. Conclusions

The properties of the micromechanical tunneling transistor as a frequency multiplier have been studied theoretically. It was found that the exponential dependence of the tunneling current on the tip-contact gap produces a sharp current rise when the tip comes within a few Angstroms of the contact. This, in turn, manifests as an output spectrum rich in harmonics. In particular, the amplitude of the 42nd harmonic is 10% down from the fundamental. On the other hand, the low current typical of tunneling processes would dictate the use of multiple tips (tip arrays) to achieve larger output currents. The XMMTT has potential for a new "electronics" technology based on nanoelectromechanical principles, which might have application in low-power and radiation-tolerant systems.

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